THE PURPOSE AND THE CONTENT OF A COURSE OF MATHEMATICS FOR TECHNICAL AND MANUAL TRAINING HIGH SCHOOLS.

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Introduction.

In the treatment of this subject I shall confine myself to those schools only that require a grammar school diploma, or its equivalent, for admission, that have a four year's course, and that are manual training or technical high schools as distinct from the "trade" school. I have confined my investigation, mainly, to public high schools, for with the institutions supported by the public the problem is more vital, and demands the attention of the public school teacher to a far greater extent than that of private institutions.

These limitations make the field which I am to cover differ much from the field covered in the Report on Mathematics in Technical Secondary Schools as compiled for the International Commission on the Teaching of Mathematics, for, not only will my discussion be along narrower lines, but I intend also to propose a course of mathematics, which seems to me to meet the demands of the times for such schools.

To this end, in order to get information concerning present actual conditions in manual training or technical high schools, or in high schools having manual training departments, I sent out a list of questions to a hundred of the largest public high schools in the United States, coming under one or another of the above heads.

From about seventy of these schools, representing twenty-eight states, I received more or less complete answers, of which twenty-five returned me answers quite complete, including such representative schools as: the Stuyvesant High School of New York City, the Rindge Technical High School of Cambridge, the Richard T. Crane Technical High School of Chicago, the Hughes High School of Cincinnati, the Denver Manual Training

High School, the Westport High School of Kansas City, Mo., the Minneapolis South High School, the New Haven High School, the North East High School and Southern Manual Training High School of Philadelphia, the Boardman High School of Seattle, the Springfield, Mass., Technical High School, and the Mechanic Arts High School of Boston.

The list of questions follows:

1. What was the total enrolment to your school last year?

2. How many were taking manual training?

3. If your course is a four years' course, of the class entering four years ago, the class of 1912, how many elected manual training?

The following questions are to be answered with reference to those who elected manual training?

4. At the beginning of the second year, how many returned as members of the class of 1912?

5. At the beginning of the third year, how many returned as members of the class of 1912?

6. How many were graduated last June?

7. How many of the graduates have entered, or expect to enter, some higher institution?

8. How many years of mathematics are required of the manual training pupils?

9. What branch of mathematics is taught the first year? *a.* Periods per week? *b.* Text?

10. What branch of mathematics is taught the second year? *a.* Periods per week? *b.* Text?

11. Comment upon other courses in mathematics offered to manual training pupils.

12. Is any effort being made to correlate the different branches of mathematics?

13. If so, to what extent?

14. Is it the policy of the school to encourage, or to discourage, preparation in the manual training course for higher institutions?

15. Is your school coeducational?

16. If so, is the same mathematics course required of both boys and girls?

17. Remarks.

The purpose of the first two questions was to learn the size of the school and the proportion of the pupils taking manual training, in order to judge the value of the answers that follow, hence, no more comment is needed concerning them.

Furthermore, the results of the last two questions, 15 and 16,

I will discard as useless, as I found that, in all schools that are coeducational, practically the same mathematics is offered for the girls as for the boys, excepting that in most cases less is required of the girls.

The results of questions 7 through 14 I will discuss as needed, but questions 3 to 6, inclusive, which deal with the falling off in attendance from year to year, show a condition that requires most serious attention.

By averaging the statistics of the twenty-five schools from which I received answers to these four questions, I find that, of every hundred pupils who entered the lowest class in 1908, but sixty-three returned the second year, but forty returned the third year, and but twenty-seven were graduated. The decrease is therefore far greater in the first two years than in the last two years of the course.

Hence, noting that out of every hundred pupils sixty leave during the first two years, my problem is to propose a course of mathematics: (*A*) that will give as much power as possible to those pupils who can remain two years or less, which I shall call *short time pupils;* (*B*) that will furnish for those who *can* remain throughout the course an incentive to complete the four years—*possible graduates;* (*C*) that will render it possible for those desiring to enter some higher institution, to be able to do so—*college preparatory pupils.*

A. Short Time Pupils.

Of these there are two classes: (*a*) Those who lack the ability to perform the work of high school grade, most of them older than the average and delinquent; (*b*) those who, because of economic conditions, must leave school to go to work.

For group (*a*): inasmuch as it is not the function of the school to supply ability, and as this group, up to the present, has not shown evidences of ability to acquire more than has already been acquired in the grammar schools, I shall not consider its special needs in my discussion, believing that the few who have not already been directed to trade schools should be allowed to get what little they can from the course designed for the more capable ones.

For group (*b*): first, what are the ends we should aim to at-

tain? To me the following seem most important: power to think clearly, habit of doing work accurately, rapidly and neatly, and spirit of self-reliance; also, for those who can stay the major part of the two years, ready skill in the control of necessary mathematical ideas to forward the technical problems in hand, habit of functional thinking, acquaintance with computing methods and significant figures, and methods of minimizing labor in calculations.

The means for securing the above ends I shall consider under the following heads: I. Arithmetic, II. Algebra, III. Geometry, IV. Trigonometry and Tables, V. Applied Problems, VI. Correlation.

I. *Arithmetic.*—It is true, that, in general, the *short time pupils* will have actual use for no mathmetics beyond the simpler calculations in arithmetic. Hence, it is my conviction that arithmetic should occupy an important place in the course for these pupils. First of all, it should serve to clinch the work in arithmetic already given in the grammar schools; then, through substitution in formulas, it should lead the way to algebraic forms; through graphical tabulations, lead the way to negative numbers; and, through mensuration, lead the way to a fuller acquaintance with the properties of geometrical figures. At no time in the course should there be an extended period, when practice in arithmetical computations should be lacking.

II. *Algebra.*—First, let us consider the value of training in algebra. Prof. Hedrick, of the University of Missouri, in an address before the National Educational Association in 1910, said: "The symbolism of algebra has direct value, for we can express in brief forms the facts of science and everyday life, and work with these forms by abbreviated processes. Algebra is made up of operations upon relations between quantities—the study of variable quantities, relations between these variable quantities, and acquisition of ability to control and interpret such relations. Now the very essence of science is to discover the precise dependence of changes in one quantity upon changes in another, which Prof. Klein calls 'habit of functional thinking.' The spirit of algebra is the dependence of varying quantities upon each other—cause and effect."

Furthermore, in the Report of the Committee on Mathematics

in the Public and Private Secondary Schools of the United States is found the statement that the general idea underlying the teaching of mathematics is the development of the scientific aspect.

Hence, if we are to give our *short time pupils* any training in pure mathematics—and, under the head of Applied Problems, I shall show that we should—I claim that this training should be, in the main, with algebraic processes.

The results of my investigation will show to what extent this training is being given. Of the fifty-eight schools answering question 9 concerning the mathematics for the first year, fifty-two offer algebra, three algebra and arithmetic, and three offer algebra and geometry, using as a text Short and Elson's "First Year Mathematics." The texts are distributed as follows:

Hawkes, Luby and Touton	12
Wells' Essentials or Academic	15
Wentworth	9
Milne	10
Young and Jackson	4
Slaught and Lennes	4
Marsh	4
Hull	1
Hart	1
Stone and Millis	1

Now, as I consider this array of texts I am convinced that over-emphasis is placed upon theory. Much of the work now found in these texts should be omitted, not only for the first year, but for the first two years, and it should be replaced by problems connected more closely with allied subjects. Modern industrialism, with its demand for tangible success, has led to a great out-cry for more practical school work, and has registered its contempt for "mere theory." We must give work to the pupil that is as practical as possible, also that is at the same time interesting to him. Prof. Bennett, of Columbia, says that even the cultural value of a subject depends upon the extent to which that subject can be, and actually is, linked with the activities and thought content of real life; that in this way the old static, historic, idealistic conception of mental discipline is being replaced by a dynamic, realistic, practical view.

The value and place of the practical problem will be discussed at greater length under the head of Applied Problems.

III. *Geometry.*—The fact that the catalogs of all "trade" and "industrial" schools, to which I have had access, require a knowledge of many of the properties of geometrical figures, alone gives that subject a place in the course. The pupil should have drill in the mensuration of plane and solid figures, should know how to construct many of the plane figures in common use, and through applications should become acquainted with many other properties of geometrical figures besides areas and volumes, such as those of congruent and similar figures. In an informal way he should be led to see the connections between many of the statements of geometry, in order to aid his memory of them and to give him some feeling of security as to the truth of the facts.

Now, as to the actual conditions at the present time, I received answers to question 10 from fifty-eight schools. Of these, forty-eight offer plane geometry alone; six offer algebra and geometry —of these six, three use Short and Elson's "Second Year Mathematics," and the other three offer parallel courses; two schools continue algebra and arithmetic, offering plane geometry the third year—the Detroit Central High School, and the Beloit, Wisconsin, High School; and two offer plane geometry and shop mathematics—the Newark Technical School, and the New Haven High School.

The texts for geometry are distributed as follows:

Wentworth and Smith	10
Wells	7
Shutts	6
Stone and Millis	3
Milne	3
Phillips and Fisher	2
Durell	2
Saunders	2
Shultze and Sevenoak	2
Slaught and Lennes	1
Bush and Clark	1
Robbins	1
Lyman	1

Noting these courses offered, it is evident that great emphasis

is laid upon formal geometry. I consider this a very weak point, for formal geometry, whatever may be its value in developing the "logical faculty"—and this is denied by some educators—is a very difficult subject even for the best pupils, and is to many nothing else than an exercise for the memory. In this connection I quote from Prof. Young's address before the Association of Mathematical Teachers in New England: "Formal methods of reasoning with continued explicit reference back to a hypothesis is not the natural method of reasoning to high school pupils. . . . A really formal proof in a high school course is utterly impossible." These views, expressed with reference to formal geometry for all high school pupils including even the pupils preparing for college, apply even more strongly to pupils in technical schools. Hence, I must conclude that a course of such doubtful value as formal geometry has no place whatever in the course of mathematics for the *short time pupils*—these pupils, many of whom must because of economic conditions go out so soon to begin the battle of life.

IV. *Trigonometry and Tables.*—For practice in accuracy and rapidity of arithmetical computations trigonometry offers a good field. Moreover, it is useful in impressing upon the pupil the idea of ratio, and the equality of ratios of corresponding sides in similar figures. Experience in class rooms with low grade divisions has shown me that the pupils comprehend trigonometry much more quickly than they do formal geometry. Again, it offers a field for many interesting and useful problems.

The use of tables in the course is for the direct purpose of giving the pupil some acquaintance with methods of computation, by which he may minimize his labor. He should be taught to use the square root and cube root tables, logarithmic tables to four places and trigonometric tables to four places, and he should be taught to what extent results obtained by the use of these tables are of practical value.

V. *Applied Problems.*—Under the head of Algebra I have already brought out the over-emphasis on the theoretical, and the demand of the industrial world for the practical problem. Recognizing this demand for problems from real life, as one must, I will now consider the kind of problem needed. I doubt very much if problems about areas of states, about populations,

planet and star distances, imports and exports, latitudes and longitudes, linoleums and ornamental designs, etc., are any more real, or of much more practical value, to the boy than the much condemned "cistern," "work," and "hare and hound" problems. Few of such problems have a place, but the real applied problem should be, I believe, a problem that might reasonably occur in the pupil's own actual life. Many of this sort should be brought into the early part of the high school course to prepare the pupil, soon to leave, to do real work of the kind similar to the problems learned. Continued use of mathematics in real problems gives the pupil, first of all, an idea that mathematics can be used, and, also, develops some judgment as to how to use it. He will come to regard mathematics as a valuable tool that can do efficient work.

Because of the enthusiasm for, and the evident demand for, real problems to the exclusion of theory, there has arisen in England the Perry Movement, and with it the neglect of pure mathematics. At the convention of the National Educational Association in 1910, Mr. William Breckenridge, of the Stuyvesant High School, New York City, speaking on "The Perry Movement," said: "We must keep in mind that pure mathematics is, after all, the thing the student much needs. Real problems are unorganized and ungraded, so that the study of them leads nowhere. The student is hurried through a mass of problems and finds on their completion that the product of the work is a confused notion of everything in general and no clear idea of anything." Further than that, that many schools, which have complete courses for training for the industries, see the need of pure mathematics is evident from quotations from their catalogs. In that of the Newark Technical School is the following: "From an educational standpoint the study of mathematics is essential because of its disciplinary value in forming habits of attention, of concentration, of accuracy and precision of thought. . . . It is equally necessary from the economic and practical standpoint. Without a knowledge of mathematics but little, if any, progress can be made in any branch of physical science. . . . Men with a knowledge of mathematics are needed in the machine shop, in the draughting room, and in the field."

In the catalog of the David Rankin, Jr., School of Mechan-

ical Trades, St. Louis, Mo., is found the requirement that all day students shall take courses in arithmetic, elementary geometry, formulas and solution of problems involving one or two unknowns, mechanics, and elements of trigonometry.

Some reformers are urging that the mathematics of the technical schools should follow the lines of the Perry idea, a condition which already exists in many of the industrial and trade schools. To show to what extent this is being done in certain industrial schools, and at the same time to show of how little value such methods of training are to the student, I quote the following from the Supplementary Report on the Industrial Schools of Secondary and Intermediate Grade for the International Commission: "In certain industrial schools mathematics, though definitely included in the course, is not taught as a separate and distinct subject, but introduced as the student strikes some phase of the work in the shops or draughting rooms requiring knowledge of a certain fact, which is brought out for immediate use. In this way he obtains his mathematics. . . . In the first place, such instruction can hardly develop originality on the part of the pupil, and, in the second place, he has not the apperceptive mass from which various mathematical facts and relations can be drawn out. At best he can only be made to see that the statements made to him are plausible. At most he sees only a glimmer of light, and then comes total darkness. When one realizes how difficult to most pupils are certain propositions in geometry, what must be one's judgment upon a method of teaching which tries to impress isolated mathematical facts upon the mind—upon a mind, moreover, which has not been prepared by constant drill to recognize intuitively mathematical relations? The conclusion must be that such teaching can not give the pupil power or lasting knowledge."

If such is the conviction of the committee concerning the purely industrial schools investigated by them, certainly pure mathematics has a place in the technical high school. Although present conditions in these schools show that too great emphasis is placed upon pure mathematics, and although, on the other hand, there are those who would eliminate pure mathematics as such from the course entirely, I contend that a reasonable middle course can be and should be made.

VI. *Correlation.*—To gain ready skill in the control of necessary mathematical ideas, it appears to me impossible to treat the subjects named as separate sciences, but such parts of arithmetic, algebra, geometry and trigonometry must be selected as are fundamental for such an end. Mathematics should not be a series of discrete subjects, each in turn to be studied and dropped without reference to the others or to mathematical problems that arise.

In my investigation I found that, of fifty schools answering questions 12 and 13, eighteen made no effort whatever at correlation, twenty made but slight effort, and twelve made much more effort in that direction, but practically none correlated the work during the first two years, excepting the three using Short and Elson's "First and Second Year Mathematics," and the Hughes High School of Cincinnati.

As I consider the ends for which I believe we should aim for the *short time pupils,* I do not accept as a valuable means the fusion of algebra and formal geometry. I have already shown why I do not believe that formal geometry has a place in the course for these pupils. Furthermore, concerning the fusion of algebra and formal geometry, Prof. J. W. A. Young, of Chicago, speaks to the point when he says: "It seems to me that the fields of algebra and (formal) geometry are essentially different, both in ground covered and in methods used. These differences seem sufficient to preclude the possibility of the fusion of the two into a homogeneous whole that shall be neither algebra nor geometry, but a real composite of the two."

That many leading educators in this country are seriously considering the need of correlation of the different branches of mathematics in the secondary schools, there is but little doubt. We need to break down the watertight compartment idea of mathematics being made up of a number of isolated and unrelated subjects. We need to unify more fully all our mathematics."

My discussion, up to this point, has shown that, in general, the mathematical courses in technical and manual training high schools in the United States differ in no important educational feature from those in the classical high schools, that the courses are designed, not primarily for the majority of the pupils enter-

ing the school, but for the group of pupils using the high school as a step towards college preparation. I have shown that to meet the needs of this larger group, the *short time pupils,* certain modifications of the mathematical courses are necessary. For this purpose I have considered the values of arithmetic, of algebra, of geometry, and of trigonometry. I have shown the need of problems from real life, bringing out the danger of over-emphasis in this direction. I have shown the need of training in pure mathematics. I have shown that formal geometry has no place in the course. Finally, I have shown that, in order to get the greatest value out of the course for the first two years, the different branches should be combined into a single course of mathematics.

B. Possible Graduates; Those Who Can Stay if They Will.

The main purpose for the first two years in reference to this group is to have the course of such nature as to furnish a strong incentive to remain in the school. It is obvious that the work should be simplified and adapted to the ability of the immature pupil of the first year, so that he will not become discouraged at the beginning of his high school course. It is also obvious that the subject matter should arouse the interest of the pupil, and that he should feel that it is of such a nature as to be of value in meeting his needs. In an address by Prof. W. S. Munro, of the University of Missouri, I find the following: "Unless a pupil feels that there is some reason for studying a subject, unless a pupil has a motive for studying a subject, our efforts to teach that subject are practically fruitless. The only satisfactory motive for the study of mathematics in the first year or two of the high school must be based upon, or connected with, a feeling that the subject matter of mathematics is worth while or valuable in itself, that it is useful, good for something." Hence, the pupil's course should be vitalized by many problems taken from actual conditions about him. He should not be called upon to master a collection of abstract truths, to master the fine distinctions of a logical demonstration, which is so ill-suited to his powers, when the reason for so doing is that he

may gain only a promised but shadowy mental discipline. The belief that mathematics is of real worth should come to the pupil early, for which reason he should see as much of the field of mathematics with its applications as possible. The subject matter should be so arranged and problems of such sort that he may see in his different courses some unity, see the connection of the different branches of mathematics with each other, with his drawing, with his science, and with his shop practice.

As a means of saving time, of decreasing labor in calculations, use of square root, cube root, and logarithmic tables will commend itself to the pupil.

If the course of mathematics for the *possible graduates* for the first two years is based upon the above ideas, a strong incentive will thus be offered them to remain.

Before considering the content of the course for these pupils for the last two years, we must have before us the purpose of the technical high school. From the Report of the Committee on Industrial and Technical Education in Secondary Schools, presented at the meeting of the National Educational Association in 1910, we find the following: "A technical high school has for its distinct purpose the preparation for industrial leadership, positions requiring skill and technical knowledge. The instruction deals not only in important manual operations, but also with principles of science and mathematics and their direct application to industrial work." In another part of the report we find: "Its (the secondary technical school) main object is the preparation of pupils for efficiency in a large group of important positions in industrial life, aiming to cultivate industrial intelligence, these qualities essential for efficient industrial leadership rather than abstract reasoning power." Hence arises the need that a pupil learn how to attack a specific problem, how to analyse it, and how to select the necessary tools for solving it.

Now, let us note what the schools investigated are doing in order to gain such results. Of the fifty schools answering the question 11 concerning the mathematical courses offered beyond the second year, thirty-six named college preparatory subjects only, that is, solid geometry, trigonometry, and advanced algebra—of these only nine claimed to attempt correlation to any degree; two schools offered commercial arithmetic; three

offered analytical geometry, three surveying, and six a definite course in shop problems.

From these statistics it is evident that by far the majority of these schools are offering not much else than the course laid out for them by the colleges. Here again, I am convinced that undue emphasis is placed upon pure mathematics, to the exclusion of applied work. I would continue work in pure mathematics for the valid reasons given before, but I would replace a large share of it by a definite course in applied mathematics. This course should include problems from physics, from chemistry, from the drawing department, and from the shops.

As to the nature of the shop problem, if it is not taken directly from the shops, it should, at least, fit the conditions existing in the shops; it should be stated in terms of the shop as far as practicable; it should furnish results approximated to a required degree of accuracy that represent good shop usage; and it should not be made unnecessarily complicated so as to make solution more difficult.

Part of this course of applied mathematics should be devoted to acquainting the pupil with the use of surveying instruments. Much practice should be required in the use of different kinds of tables and of other time saving devices.

In discussing the course for the *short time pupils,* my aim was to develop the power of thinking clearly, habits of accuracy, rapidity and neatness, habit of functional thinking, ready skill in the control of necessary ideas for meeting the problems of life requiring the use of mathematics. The course for the *possible graduates* should be of such nature as to continue to develop these same ends.

C. College Preparatory Pupils.

For these pupils the problem is not what mathematics shall be offered, for that is determined by the higher institutions, but, first, why pupils should be allowed to prepare for college in technical high schools, and, secondly, in view of the course that I would offer the first two years, how the course of the last two years could be arranged to meet college requirements.

I. From the Report of the Committee on Industrial Education before the National Educational Association in 1910, I

quote the following: "Many educators feel that no system of education should be allowed to develop blind alleys, and wish to see the way kept clear for any youth to pass from one school to the next higher." Again, from the Report of the Committee of Nine on the Articulation of the High School and College before the National Educational Association in 1911, I quote the following: "The high school period is a testing time, a time for trying out different powers, a time for forming life purposes. The opportunity should be provided for the student to test his capacity in a fairly large number of relatively diverse kinds of work. In the high school the boy or girl may very properly make a start along the line of his chosen vocation, but a final choice should not be forced upon him at the beginning of that career. If he makes a provisional choice early in the course, there should be ample opportunity for readjustment later in the high school."

From another part of the same report, in order to show the value of the technical school training for those who may continue school education, I find this: "The organic conception of education demands the early introduction of training for individual usefulness, thereby blending the liberal with the vocational, for only then does liberal education receive its social significance and importance. The boy who pursues both the liberal and the vocational sees the relation of his own work to the work of others and to the welfare of society."

Hence, it is seen that many leading educators believe that, after the high school course is begun, there should still exist the opportunity to go on to institutions beyond the high school, and that the training received in such schools as technical high schools is of distinct value to such pupils.

In addition to this belief, there is a demand for such opportunity, evidenced by the fact that so many of the graduates of technical and manual training high schools do enter higher institutions. From the thirty-five schools answering question 7 in my list of questions, I find that 35 per cent. of the graduates, more than a third, enter higher institutions. This is one per cent. higher than the average for all public high schools in the United States, as found in the Report of the Commissioner of Education, for the year 1910. I find, too, that, of the fifty-six

schools answering question 14, it was the policy of forty-two to encourage preparation for college in the manual training departments.

Many a pupil does not find himself for some time after he enters high school, for it is then that he is just entering upon the period of adolescence, and, as the high school subjects are, in the main, new to him, it is only by the actual study of them that he can know whether he will find them interesting to him and whether he can master them.

Before the Association of Mathematical Teachers in New England, at the December meeting in 1912, Prof. D. E. Smith said that we must keep an open door in mathematics, because we know not what mathematicians may be among our numbers, so I claim we must keep the open door in all high schools for higher institutions, for we know not what scholars may be among us.

II. Assuming that we do recognize the necessity of allowing pupils entering the technical high school to prepare for college, now comes the problem as to how to arrange the course to make it possible.

As I have planned the work in mathematics, and have held in mind the remaining subjects of the course for the first two years, the college preparatory pupils, in general, have not been considered. Still, the work in English, in elementary science, in history, with the probability of one year of a modern language, and in mathematics would serve as a very good foundation upon which to build.

In mathematics, the only problem to offer a real new difficulty would be the formal demonstration in geometry, for the pupil should have been well informed about most of the properties of geometrical figures, and should have had enough theoretical work in algebra to serve as an equivalent of the work usually done in algebra in the first year of the average high school.

At the beginning of the third year should come the time for the entire separation of these pupils from the *possible graduates,* for, by that time, most pupils will have come to know which course is most fitting for them.

Let us suppose that a school has a program providing for thirty periods a week. For the third year, I would divide it

as follows: Mathematics, 5; Modern Language, 5; English, 5; History, 3; Physics, 5; with an elective, if desired, in the other Modern Language, Drawing, or Shop Practice, 5 or 6. For the fourth year, I would make the following division: Mathematics, 5; Modern Language, 5; English, 5; History, 3; Chemistry, 5; with an elective, if desired, in the other Modern Language, Drawing, or Shop Practice, 5 or 6.

Based upon the requirements I have laid down in the preceding discussion I will now propose briefly what seems to me a reasonable content for the course of mathematics for the schools discussed:

A. First Year.

I. Plane figures. Mensuration of those familiarly known to the pupil, giving practice in use of fractions and mixed numbers. Board measure.

II. Construction of plane figures, using for instruments compass, ruler and protractor. Drawing to scale of plans for models and of polygons.

III. Square root. Formulas from geometry and the sciences.

IV. Addition and subtraction of algebraic expressions. Graphs of statistics. Problems from geometry.

V. Equations. Parentheses. Problems.

VI. Multiplication of algebraic expressions, special products, equations, problems.

VII. Division using monomial and binomial divisors only. Equations.

VIII. Simultaneous linear equations. Problems. Graphs.

IX. Indexed list of definitions and principles of geometry used, also of formulas.

B. Second Year.

I. Review of algebraic operations.

II. Factors:

(*a*) Monomial factors.

(*b*) Binomials—difference of two squares, sum or difference of two cubes.

(*c*) Trinomials—squares and cross products.

(*d*) Polynomials of four terms containing a common binomial factor.

III. Quadratics having rational roots, both of one and two variables, solved by factoring. Problems.
IV. Common factors, common multiples, fractions, equations.
V. Roots, use of square root and cube root tables. Formulas. Radicals containing monomial quadratic surds only.
VI. Quadratics having irrational roots, solved by completing the square or by formula. Problems.
VII. Ratio and proportion. Similar polygons. Trigonometric functions of acute angle of a right triangle. Similar solids.
VIII. Exponents and logarithms. Only the treatment of exponents necessary for intelligible use of logarithms.
IX. Trigonometry. Solution of right and oblique triangles.
X. Indexed list of formulas and of principles of geometry used. Square root, cube root, logarithmic, and trigonometric tables.

C. Third and Fourth Years.

I. *Possible Graduates.*—Since the course for those who finish their formal education in the high school should be as closely correlated as possible with the work of the shops, drawing room, and science laboratory, I shall make no attempt to fix the order in which the work should be taken up, as that would depend much upon the order in which the subjects are taken in the other departments.

I believe that the course should be divided into two rather distinct parts, one part consisting of pure mathematics mainly, and the other part a course in shop problems, alternating with the first, if possible. For the first part, trigonometry with surveying, followed by algebra, should be offered. For the second part, there should be problems on simple machines, problems on weights of bars of various shapes and materials, on strength of materials, problems on horse power of engines, motors, belting and shafting, problems on hoists, milling machines, hand and engine lathes, lathe indicators, safety valves, heating surfaces of boilers, etc. For the solution of these problems the pupils should be taught the use of the slide rule and of all tables that

will minimize labor. In other words, the work in mathematics should be both pure and practical, the practical being as closely connected with the work of the mechanical departments as possible.

II. *College Preparatory Pupils.*—During the third year formal plane geometry should be given and completed so that the pupil will be prepared to take preliminary examination at the end of that year.

During the fourth year the course should include solid geometry for about twelve weeks and algebra for the remaining time, covering the college requirements for advanced algebra.

At the present time one radical change in our educational system that is being advocated by many prominent educators is the division of the first twelve years of the school course into two periods of six years each. This change has the support of such a well-known educator as Prof. David Eugene Smith, of Columbia, and also was urged in the Report of the Committee on Economy of Time in Education before the National Educational Association in 1911.

Another radical change that is being proposed is the raising of the compulsory attendance age limit from fourteen years to sixteen or eighteen years. Dr. Franklin H. Dyer, Superintendent of Schools, Boston, Mass., is a strong advocate of this change, but along with it he is advocating part time schools, especially for pupils beyond fourteen years.

Now the course I have outlined would, I believe, fit very well into either or both of these schemes, because much emphasis is placed upon constant drill in arithmetical computation, because the course is continuous, and because the subject matter has been so selected as to make the mathematics worth while for the solution of problems likely to be met in everyday life.

The experiment with the part time school, begun by Superintendent Dyer, has been tried, at some length, in the Hughes High School of Cincinnati, with apparent success. Along with it has been developed a course of mathematics for these pupils that is in many respects similar to the one proposed above for all pupils in the technical high schools.

In conclusion, viewing the many problems at present needing

solution in the technical and manual training high schools, I contend that if the mathematical course for these schools follows along the lines advocated above, they will be accomplishing to far greater extent than at present what they should for the best interests of the pupils, hence, will be giving the coming generation a training that will make them far better and more efficient citizens.

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